BENHA UNIVERSITY
FACULTY OF ENGINEERING (SHOUBRA)
ELECTRONICS AND COMMUNICATIONS ENGINEERING

ECE 211

Measurements and Instrumentations

(2022 - 2023) 1st term

Lecture 2: Measurement Errors (part2).

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Chapter 2: Measurement Errors

2.3 Measurements Characteristics:

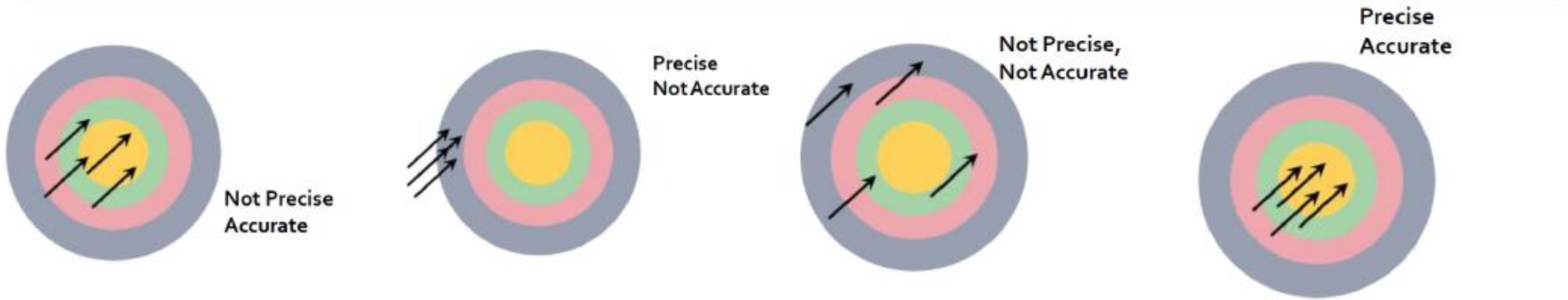
Accuracy and Precision:

1. Accuracy

Accuracy is defined as the degree of **closeness** of a measured value compared to the true value of the quantity to be measured.

2. Precision

Precision is defined as the degree of similarity of repeated measurements.



2.3 Measurements Characteristics:

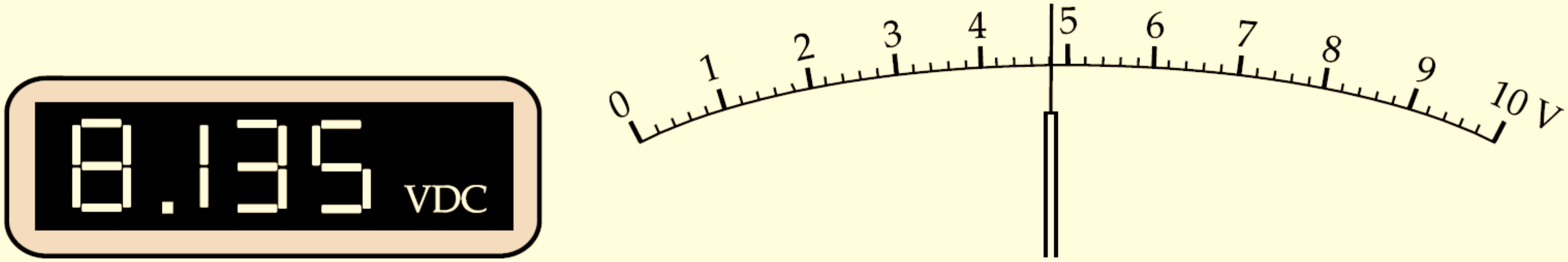
Resolution and Significant Figure:

3. Resolution

Resolution is defined as the **smallest change** in the measured quantity to which an instrument will respond.

4. Significant Figure

Significant Figure is defined as **the number of digits used** to represent a measured value. The more the number of significant figures, the more precise is the quantity.



(a) Digital voltmeter display with a 1 mV precision

(b) Analog instrument display with a 50 mV precision

Figure 2-4 Measurement precision depends on the smallest change that can be observed in the measured quantity. A 1 mV change will be indicated on the digital voltmeter display above when the quantity changes by ±0.001 V. For the analog instrument, ±50 mV is the smallest change that can be noted.

Chapter 2: Measurement Errors

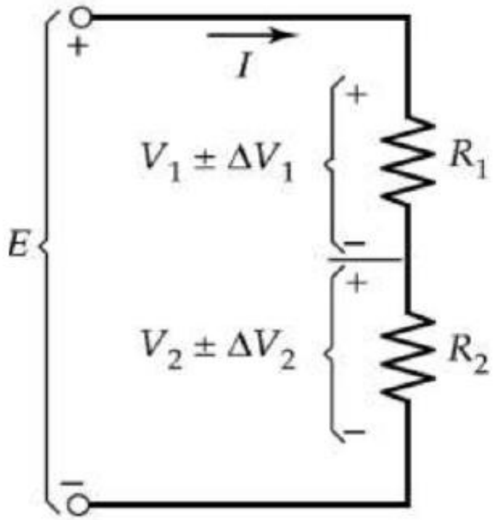
1. Types of Measurement Errors.
2. Absolute and Relative Errors.
3. Measurements Characteristics.
4. Measurement Error Combinations.

2.4 Measurement Error Combinations:

When a quantity is calculated from measurements made on two (or more) instruments , it must be assumed that the errors due to instrument inaccuracy combine in worst possible way. The resulting error is then larger than the error in any one instrument.

2.4 Measurement Error Combinations:

1. Errors in Sum of quantities

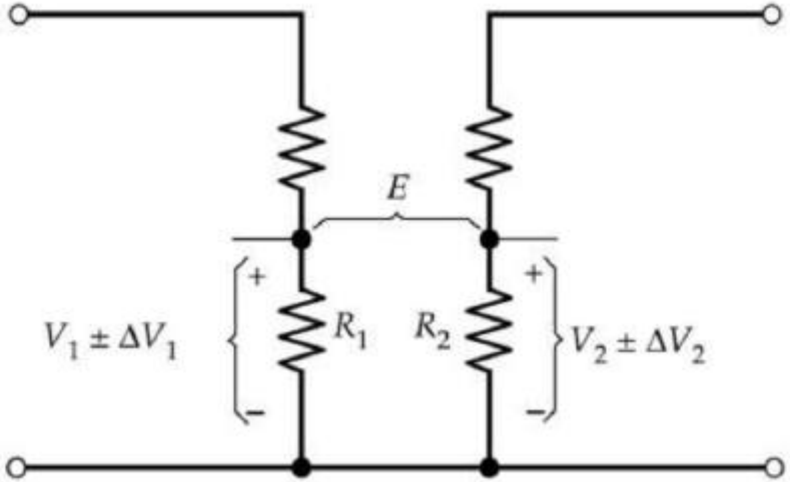


$$
\begin{aligned}
E &= V_1 + V_2 \\
&= (V_1 \pm \Delta V_1) + (V_2 \pm \Delta V_2) \\
&= (V_1 + V_2) \pm (\Delta V_1 + \Delta V_2)
\end{aligned}
$$

Error in Sum

Error in the sum of quantities equals the sum of absolute errors.

2. Errors in Difference of quantities



$$
\begin{aligned}
E &= V_1 - V_2 \\
&= (V_1 \pm \Delta V_1) - (V_2 \pm \Delta V_2) \\
&= (V_1 - V_2) \pm (\Delta V_1 + \Delta V_2)
\end{aligned}
$$

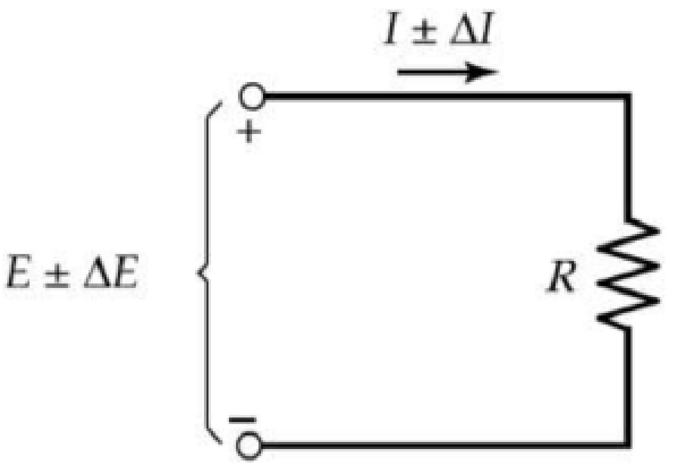
Error in Difference

Error in the difference of quantities equals the sum of absolute errors.

2.4 Measurement Error Combinations:

3. Errors in Product of quantities

$$
\begin{aligned}
P = EI &= (E \pm \Delta E) \times (I \pm \Delta I) \\
&= E.I \pm E.\Delta I \pm I.\Delta E \pm \Delta E.\Delta I \\
&\approx E.I \pm E.\Delta I \pm I.\Delta E \quad (\Delta E.\Delta I \text{ is very small})
\end{aligned}
$$



$P = (E \pm \Delta E) \times (I \pm \Delta I)$

Percentage error in P is

$$
\begin{aligned}
\%P &= \frac{E.\Delta I + I.\Delta E}{E.I} \times 100\% \\
&= \left(\frac{\Delta I}{I} + \frac{\Delta E}{E}\right) \times 100\% \\
&= (\% \text{ error in } I) + (\% \text{ error in } E)
\end{aligned}
$$

Percentage error in the product of quantities equals the sum of percentage errors

2.4 Measurement Error Combinations

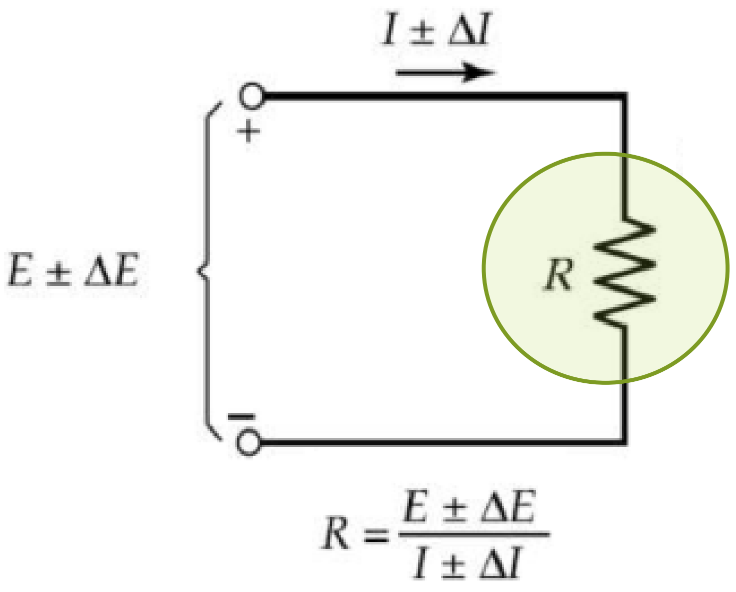
4. Errors in Quotient of quantities

$$R = \frac{E \pm \Delta E}{I \pm \Delta I}$$

Percentage error in R is

$$\%\text{error in } R = (\%\text{ error in } I) + (\%\text{ error in } E)$$

Percentage error in the quotient of quantities equals the sum of percentage errors



2.4 Measurement Error Combinations

5. Errors in quantity raised to a power:

$$P = I^2R$$

Quantity raised to a power:

%error in $A^B = B$(% error in A)

Summary

For $X = A \pm B$, error in $X = \pm$ [(error in A) + (error in B)]

For $X = AB$, % error in $X = \pm$ [(% error in A) + (% error in B)]

For $X = A/B$, % error in $X = \pm$ [(% error in A) + (% error in B)]

For $X = A^B$, % error in $X = \pm B$(% error in A)

Example 1:

Calculate the maximum percentage error in the sum of two voltage measurements when $V_1 = 100\text{ V} \pm 1\%$ and $V_2 = 80\text{ V} \pm 5\%$.

Solution

$$V_1 = 100\text{ V} \pm 1\%$$
$$= 100\text{ V} \pm 1\text{ V}$$

$$V_2 = 80\text{ V} \pm 5\%$$
$$= 80\text{ V} \pm 4\text{ V}$$

$$E = V_1 + V_2$$
$$= (100\text{ V} \pm 1\text{ V}) + (80\text{ V} \pm 4\text{ V})$$
$$= 180\text{ V} \pm (1\text{ V} + 4\text{ V})$$
$$= 180\text{ V} \pm 5\text{ V}$$
$$= 180\text{ V} \pm 2.8\%$$

Example 2:

Calculate the maximum percentage error in the difference of two measured voltages when $V_1 = 100 \text{ V} \pm 1\%$ and $V_2 = 80 \text{ V} \pm 5\%$.

Solution

$$\left.\begin{aligned} V_1 &= 100 \text{ V} \pm 1 \text{ V} \\ \text{and} \quad V_2 &= 80 \text{ V} \pm 4 \text{ V} \end{aligned}\right\}$$

$$\begin{aligned} E &= (100 \text{ V} \pm 1 \text{ V}) - (80 \text{ V} \pm 4 \text{ V}) \\ &= 20 \text{ V} \pm 5 \text{ V} \\ &= 20 \text{ V} \pm 25\% \end{aligned}$$

Example 3:

An 820 Ω resistance with an accuracy of ± 10% carries a current of 10 mA. The current was measured by an analog ammeter on a 25 mA range with an accuracy of ±2% of full scale. Calculate the power dissipated in the resistor, and determine the accuracy of the result.

Solution

$$P = I^2R$$

$$P = (10\ \text{mA})^2 \times 820\ \Omega$$

$$= 82\ \text{mW}$$

$$\text{error in } R = \pm 10\%$$

$$\text{error in } I = \pm 2\% \text{ of } 25\ \text{mA}$$

$$= \pm 0.5\ \text{mA}$$

$$= \frac{\pm 0.5\ \text{mA}}{10\ \text{mA}} \times 100\%$$

$$= \pm 5\%$$

$$\%\ \text{error in } I^2 = 2(\pm 5\%)$$

$$= \pm 10\%$$

$$\%\ \text{error in } P = (\%\ \text{error in } I^2) + (\%\ \text{error in } R)$$

$$= \pm(10\% + 10\%)$$

$$= \pm 20\%$$

END OF LECTURE

BEST WISHES